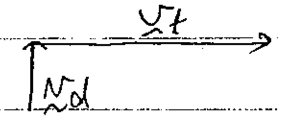
PHYS 1131 TEST 1, SESSION 2, 2005.

Q1.



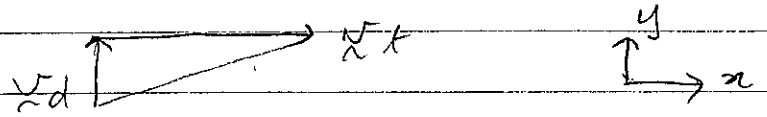
$v_d = 12\,ms^{-1}$ $\qquad v_t = 40\,km/h = 40\times10^3/(60\times60)$
$= 11\,ms^{-1}$

(a) $z - z_0 = -\frac{1}{2} g t^2$ $\qquad \Delta z = -0.25\,m$

$\Rightarrow \quad t = \sqrt{\dfrac{2(z_0 - z)}{g}}$

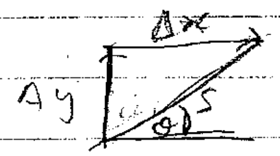
$= \sqrt{\dfrac{2 \cdot 0.25}{9.8}} = 0.23\,s$

(b) $y - y_0 = v_0 t = 12 \times 0.226 = 2.7\,m$

(c) 

$\underset{\sim}{v} = \underset{\sim}{v}_d + \underset{\sim}{v}_t = v_d \underset{\sim}{j} + v_t \underset{\sim}{i} = (12\underset{\sim}{j} + 11\underset{\sim}{i})\,ms^{-1}$

$\Delta x = v_x t = 11 \times 0.226 = 2.49\,m$
$\Delta y = v_y t = 12 \times 0.226 = 2.71\,m$

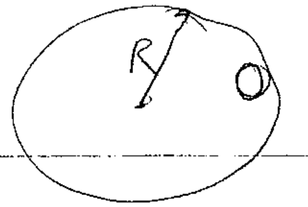


$s = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{2.49^2 + 2.71^2} = 3.68\,m$

$\theta = \tan^{-1}\left(\dfrac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\dfrac{2.71}{2.49}\right) = 47.4°$

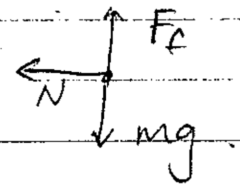
$\Rightarrow \quad s = 3.7\,m$ at an angle $47°$ from the x-axis

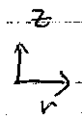
d). Yes, the flight time will be different.
Because of the horizontal acceleration, the dart will land (in general) in a different horizontal position to that found earlier. The different flight time will give a different vertical position.



2.

(a)





(b) $\sum F_z = 0$.

$F_f - mg = 0 \Rightarrow F_f = mg$

$F_f = \mu_s N$ – max. frictional force, on verge of slipping.

$\Rightarrow \mu_s N = mg$

$\mu_s = mg/N$.

(c) $\sum F_r = N = mv^2/R$

(d) $\mu_s = mg/N = \cancel{m}g/(\cancel{m}v^2)\, R = Rg/v^2$.

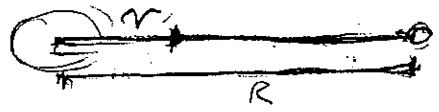
$\Rightarrow v = \sqrt{Rg/\mu_s}$

(e) $v = 7.0\,\mathrm{ms^{-1}}$.

$\Rightarrow \mu_s = Rg/v^2 = 2 \times 9.8/7^2 = 0.40$

(f) If μ_s were larger, person would still be supported

If μ_s were smaller, person would fall.



3. (a) $F_{tot} = F_e + F_m$

$$= -\frac{GmM_e}{r^2} + \frac{GmM_m}{(R-r)^2} \quad \text{in direction of moon.}$$

(b) $$\frac{GmM_e}{(r_0)^2} = \frac{GmM_m}{(R-r_0)^2}$$

$$\Rightarrow \frac{(R-r_0)^2}{r_0^2} = \frac{M_m}{M_e}$$

$$\left(\frac{R}{r_0} - 1\right) = \sqrt{\frac{M_m}{M_e}}$$

$$R/r_0 = \sqrt{\frac{M_m}{M_e}} + 1.$$

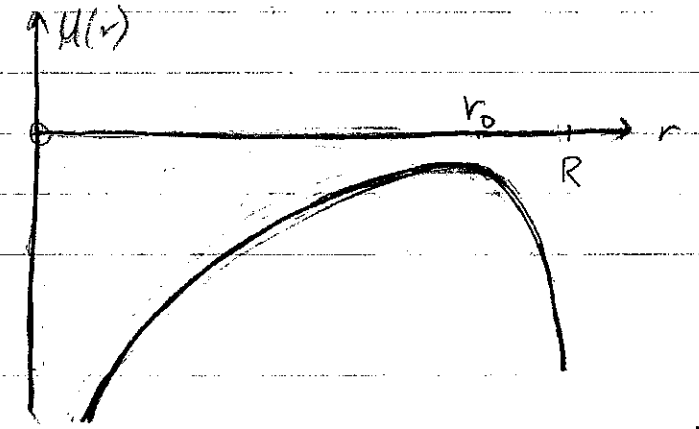
$$\Rightarrow r_0 = \left(\sqrt{\frac{M_m}{M_e}} + 1\right)^{-1} R$$

$$= \left(\frac{1}{9} + 1\right)^{-1} R$$

$$= \frac{9}{10} R.$$

(c) $$U(r) = -\frac{GmM_e}{r} - \frac{GmM_m}{(R-r)}$$

(d)



(e) $$U(r_0) = -\frac{GmM_e}{r_0} - \frac{GmM_m}{(R-r_0)} = -Gm\left(\frac{M_e}{\frac{9}{10}R} + \frac{M_m}{\frac{1}{10}R}\right)$$

$$= -\frac{Gm}{R}\left(\frac{10}{9}M_e + 10M_m\right)$$

$$= -\frac{Gm}{R}M_e\left(\frac{10}{9} + 10\cdot\frac{M_m}{M_e}\right)$$

$$= -\frac{GmM_e}{R}\left(\frac{10}{9} + 10\cdot\frac{1}{81}\right)$$

(e) cont.

$$\Rightarrow U(r_0) = -1.23 \frac{GmM_e}{R}$$

(f) minimum energy to reach moon:

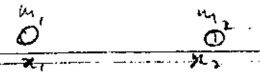
$$E = U(r_0) = -\frac{GmM_e}{R}(1.23)$$

$$E = -1.23 \frac{GmM_e}{R} = \frac{1}{2} m v_0^2 - \frac{GmM_e}{R_e}$$

$$\Rightarrow \frac{1}{2} \cancel{m} v_0^2 = \frac{GmM_e}{R_e}\left(-1.23 \underbrace{\frac{R_e}{R}}_{1/60} + 1\right)$$

$$= \frac{G\cancel{m}M_e}{R_e} \cdot 0.9795$$

$$\Rightarrow v_0 = \sqrt{\frac{2GM_e}{R_e}} \cdot 0.99$$



4.

(a) $$x_{cm} = \frac{1}{m_1+m_2}\left(m_1 x_1 + m_2 x_2\right)$$

(b) $$v_{cm} = \frac{dx_{cm}}{dt} = \frac{1}{m_1+m_2}\left(m_1 v_1 + m_2 v_2\right)$$

(c) mom. cons.

$$m_1 u_1 = m_1 v_1 + m_2 v_2 \qquad (1)$$

energy cons.

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad (2)$$

(1) : $$m_1 (u_1 - v_1) = m_2 v_2$$

(2) $$m_1 (u_1^2 - v_1^2) = m_2 v_2^2 .$$

(2)/(1)

$$\frac{\cancel{(u_1 - v_1)}(u_1 + v_1)}{\cancel{u_1 - v_1}} = v_2$$

$$v_2 = u_1 + v_1$$

(1) $$m_1 u_1 = m_1 v_1 + m_2 (u_1 + v_1) = (m_1 + m_2) v_1 + m_2 u_1 .$$

$$(m_1 - m_2) u_1 = (m_1 + m_2) v_1$$

$$\Rightarrow v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$$

$$v_2 = v_1 + u_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} + \frac{m_1 + m_2}{m_1 + m_2}\right) u_1$$

$$= \frac{2 m_1}{m_1 + m_2} u_1$$

(d) $v_{cm} = \frac{1}{m_1+m_2}(m_1 u)$

(e) $v_{cm} = \frac{1}{m_1+m_2}\left(m_1 \frac{m_1-m_2}{m_1+m_2} + m_2 \frac{2m_1}{m_1+m_2}\right) u.$

$= \frac{1}{(m_1+m_2)^2}\left(m_1^2 + m_1 m_2\right) u$

$= \frac{m_1 \cancel{(m_1+m_2)}}{(m_1+m_2)^{\cancel{2}}} u.$

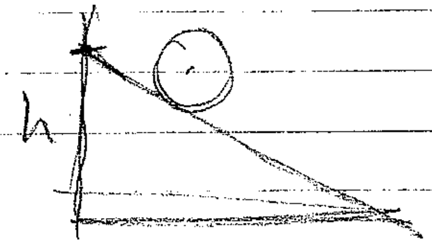
$= \frac{m_1 u}{m_1+m_2}$

(f) Yes, (d) & (e) agree. There are no external forces acting on the system,

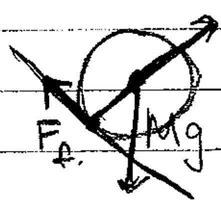
$F_{ext} = \frac{dP}{dt} = 0 \Rightarrow P \equiv \sum_i p_i = M v_{cm}$ is constant

5. (a) $dm = \sigma dA = \frac{M}{\pi R^2} 2\pi r\, dr = \frac{2M}{R^2} r\, dr.$

(b) $I = \int_0^R r^2 dm = \frac{2M}{R^2}\int_0^R r^3 dr = \frac{2M}{R^2}\frac{R^4}{4} = \frac{MR^2}{2}.$



(c)



The frictional force produces a torque about the cm.

(d) $v_{cm} = R\omega$ — rolling without slipping.

$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\frac{MR^2}{2}\cdot\left(v_{cm}/R\right)^2$

$= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}M\frac{\cancel{R^2}}{2}\frac{v_{cm}^2}{\cancel{R^2}}$

$\Rightarrow \cancel{M}gh = \frac{3}{4}\cancel{M}v_{cm}^2$

$\Rightarrow v_{cm} = \sqrt{4/3\, gh}.$

(e) $\cancel{M}gh = \frac{1}{2}\cancel{M}v_{cm}^2 \Rightarrow v_{cm} = \sqrt{2gh},$

(f) When the disc rolls without slipping all the energy is converted to translational kinetic energy & so it is faster than when the disc rolls & some of the energy is used for rot. motion

6 (a) $\omega_1^2 = k/m_1$

$\omega_2^2 = k/(m_1+m_2)$

$\frac{\omega_2^2}{\omega_1^2} = \frac{m_1}{m_1+m_2}$

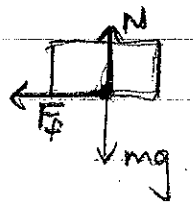
$\Rightarrow \omega_2 = \omega_1 \sqrt{\frac{m_1}{m_1+m_2}}$

$\Rightarrow f_2 = f_1 \sqrt{m_1/m_1+m_2}$

$= 2.0 \times \sqrt{\frac{2.0}{3.0}}$

$= 1.63 \quad Hz$

(b)



(c) max. $F_f = \mu_s N = ma$

$\Rightarrow a = \mu_s N/m$

$= \mu_s \cancel{m}g/\cancel{m} = \mu_s g$

$a_{max} = \omega^2 A$

$\Rightarrow \mu_s g = \omega^2 A$

$\Rightarrow A = \mu_s g/\omega^2$

$= 0.5 \times 9.8/(4\pi^2 (1.63)^2)$

$= 4.7$ cm.